Mathematical Models in Epidemiology

Fourth CI^{2}MA Workshop, Universidad de Concepción, June 18 and 19, 2018

Auditorio Alamiro Robledo, Facultad de Ciencias Físicas y Matemáticas

Organizers[1]: Raimund Bürger & Luis Miguel Villada

Programme

Monday, June 18, 2018

15.00 **Gerardo Chowell**

School of Public Health, Georgia State University, Atlanta, GA, USA

The scaling of epidemic growth in the spread of infectious diseases

The increasing use of mathematical models for epidemic forecasting has highlighted the importance of designing reliable models that capture the baseline transmission characteristics of specific pathogens and social contexts. Here, we review recent progress on modeling and characterizing early epidemic growth patterns from infectious disease outbreak data, and survey the types of mathematical formulations that are most useful for capturing a diversity of early epidemic growth profiles, ranging from sub-exponential to exponential growth dynamics. Specifically, we review mathematical models that incorporate spatial details or realistic population mixing structures, including meta-population models, individual-based network models, and simple SIR-type models that incorporate the effects of reactive behavior changes or inhomogeneous mixing. In this process, we also analyze simulation data stemming from detailed large-scale agent-based models previously designed and calibrated to study how realistic social networks and disease transmission characteristics shape early epidemic growth patterns, general transmission dynamics, and control of international disease emergencies such as the 2009 A/H1N1 influenza pandemic and the 2014-2015 Ebola epidemic in West Africa.

15.40 **Tetsuro Kobayashi**

School of Public Health, Georgia State University, Atlanta, GA, USA

To be announced

Here goes the abstract.

16.20 **Coffee break**

16.40 **Katia Vogt**

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Facultad de Ingeniería y Ciencias, Matemáticas y Estadística, Universidad Adolfo Ibáñez, Chile

Un modelo estructurado por edad-de-infección para estudiar la coinfección de VIH y VHS-2

Existe evidencia de una correlación entre la prevalencia de VHS-2– una infección viral incurable que se caracteriza por reactivación periódica del virus– y la prevalencia de VIH en la población humana. Preseántaremos un modelo matemático determinista de ecuaciones diferenciales, que modela la dinámica de la coinfecciöún de VHS-2 y VIH. Incorporamos una variable que representa edad-de-infección de VHS-2 para rastrear los perodos alternantes de infectividad de dicha enfermedad. El modelo considera relaciones heterosexuales y diferencia tres grupos poblacionales: hombres, mujeres comunes y mujeres trabajadoras sexuales. Mostraremos una expresión para el nmero reproductivo básico y el nmero reproductivo de invasión, que determinan la capacidad de invasión de una enfermedad sin y con la presencia de la otra, respectivamente. Evaluaremos el efecto de la coinfección de VIH y VHS-2 sobre la prevalencia de VIH, y tambin discutiremos el rol de los tres grupos poblacionales en la propagación del VIH.

17.20 Elvis Gavilán
CI^{2}MA & Departamento de Ingeniería Matemática, Unversidad de Concepción

Numerical solution of a spatio-temporal predator-prey model with infected prey

A spatio-temporal eco-epidemiological model is formulated by combining an available non-spatial model for predator-prey dynamics with infected prey [D. Greenhalgh and M. Haque, *Math. Meth. Appl. Sci.*, **30** (2007), 911–929] with a spatio-temporal susceptible-infective (SI)-type epidemic model of pattern formation due to diffusion [G.-Q. Sun, *Nonlinear Dynamics*, **69** (2012), 1097–1104]. It is assumed that predators exclusively eat infected prey, in agreement with the hypothesis that the infection weakens the prey and increases its susceptibility to predation. Furthermore, the movement of predators is described by a non-local convolution of the density of infected prey as proposed in [R.M. Colombo and E. Rossi, *Commun. Math. Sci.*, **13** (2015), 369–400]. The resulting convection-diffusion-reaction system of three partial differential equations for the densities of susceptible and infected prey and predators is solved by an efficient method that combines weighted essentially non-oscillatory (WENO) reconstructions and an implicit-explicit Runge-Kutta (IMEX-RK) method for time stepping. Numerical examples illustrate the formation of spatial patterns involving all three species. Future directions of research are suggested. This presentation is based on joint work with R. Bürger, G. Chowell, P. Mulet, and L.M. Villada.

20.30 **Workshop Dinner**
Restaurante Torreón, Freire 1743, Concepción

Tuesday, June 19, 2018

09.00 **Fernando D. Córdova-Lepe**
Facultad de Ciencias Básicas, Universidad Católica del Maule, Talca, Chile

An exposure model and development of infectious-contagious respiratory diseases

An epidemiological mathematical model that represents the interaction between intoxications due to pesticides and infectious-contagious respiratory diseases, with variable of interest, the application of prevention treatments for agricultural workers exposed constantly to this type of toxic, which is presented. This model is represented by a system of ordinary differential equations, which is epidemiological analyzed by obtaining and based on a reproductive number theory, complemented through numerical simulations. This presentation is based on joint work with J.P. Gutiérrez-Jara and M.T. Muñoz-Quezada.

09.40 **Aníbal Coronel**
Departamento de Ciencias Básicas, Universidad del Bío-Bío, Chillán, Chile

Some results for an inverse problem arising in a model of indirectly transmitted diseases

Here goes the abstract.

10.20 **Coffee break**

10.40 **Verónica Anaya**
Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción

A convergent finite volume scheme for an indirectly transmitted disease model

Here goes the abstract.

11.40 **Nolbert Morales**
Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción

An approximation to the minimum wave for Nicholson blowflies equation

In this work [1], we will present the approximation of traveling waves solution propagated at minumum speeds $c_0(h)$ (critical case) of the delayed Nicholson blowflies equation

$$(1)\qquad \begin{aligned} &u_t(t,x) = \Delta u(t,x) - \delta u(t,x) + pu(t-\hat{h},x)e^{-u(t-\hat{h},x)},\\ &u(t,x) \geq 0, \quad x \in \mathbb{R}^m, \end{aligned}$$

where $\hat{h} \geq 0$ and the parameters p, δ satisfy $p/\delta \in (1, e]$. In order to do that, we construct a super and sub solution to (1). Also, by that construction, an alternative proof of existence of traveling waves moving at minimum speed is given. The main difficulty in this case is due by the multiplicity of the eigenvalue associated with the linearization about 0 equilibrium, where an adequate, and different to the super-critical case, sub-solution is required. Our main theorem is

Theorem 1. *Let $p/\delta \in (1, e]$, $h \geq 0$ and $c = c_0(h)$. Then, Equation (1) possesses a traveling wave solution $u(t,x) = \phi(\nu \cdot x + ct)$. Moreover, its profile can be obtained as $\phi(t) = \lim_{n\to\infty} \phi_n(t)$, for all $t \in \mathbb{R}$, with defined by induction as follows:*

$$(2)\qquad \phi_0(t) = \overline{\phi}(t) := \begin{cases} -k_1(t-t_0)e^{\lambda_1(t-t_0)}, & \text{if } t < 0,\\ \kappa - k_2 e^{(\mu_1-\epsilon_1)t}, & \text{if } t \geq 0, \end{cases}$$

with $t_0, \epsilon_1, k_1, k_2$ defined by

$$\begin{aligned} t_0 &:= \frac{2}{\lambda_1}, \quad \epsilon_1 := \frac{-c_0 + 2\mu_1 + \sqrt{(c_0-2\mu_1)^2 + 4\ln(e\delta/p)e^{-r\mu_1}}}{2},\\ k_1 &:= \frac{\kappa\lambda_1(\epsilon_1-\mu_1)e^2}{\lambda_1 + 2(\epsilon_1-\mu_1)}, \quad k_2 := \frac{\kappa\lambda_1}{\lambda_1 + 2(\epsilon_1-\mu_1)} \end{aligned}$$

and

$$\begin{aligned} \phi_{n+1}(t) := &\frac{p}{\delta(\alpha_2-\alpha_1)}\int_{-\infty}^{t} e^{\alpha_1(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds\\ &+ \frac{p}{\delta(\alpha_2-\alpha_1)}\int_{t}^{\infty} e^{\alpha_2(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds \end{aligned}$$

for all $n \geq 0$.

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References

[1] Gomez A, Morales N. An approximation to the minimum traveling wave for the delayed diffusive Nicholson's blowflies equation, DOI: 10.1002/mma.4401

[2] Wu J, Zou X. Erratum to "Traveling wave fronts of reaction-diffusion systems with delays"[J. Dynam. Diff. Eq. 13,651,687(2001)], J. Dynam. Diff. Eq. 20 (2008) 531–532.

12.20 Helí Elorreaga Aldaz

Departamento Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción

Dynamics of a Kolmogorov-type predator-prey model with two discrete delays

In this work we consider a Kolmogorov-type predator-prey model with two discrete delays:

$$\begin{aligned} \dot{x}(t) &= x(t)f(x(t-\tau_1), y(t)), \\ \dot{y}(t) &= y(t)g(x(t), y(t-\tau_2)) \end{aligned} \tag{3}$$

Firstly, we study the absolute stability and conditional stability of the system by analyzing its associated characteristic equation. By choosing the delay as the bifurcation parameter, we show that Hopf bifurcation can occur as the delay passes through some critical values. Using the normal form theory and central manifold argument, we establish the direction and stability of Hopf bifurcation. Finally, we present an example with numerical simulations in order to verify the theoretical results obtained.

References

[1] B.D. Hassard, N.D. Kazarinoff and Y.H. Wan, Theory and Applications of Hopf Bifurcation, Cambridge University Press, Cambridge, UK, 1981.

[2] S. Ruan, Absolute stability, conditional stability and bifurcation in Kolmogorov - type predator - prey systems with discrete delays, *Quarterly of applied mathematics.*, **59** (2001), 159–173.

[3] S. Ruan and J. wei, On the zeros of transcendental functions with applications to stability of delay differential equations with two delays, *Mathematical Analysis.*, **10** (2003), 863–874.

[4] H.L. Smith, An Introduction to Delay Differential Equations with Applications to the Life Sciences, Springer. New York.